GEOMETRY.

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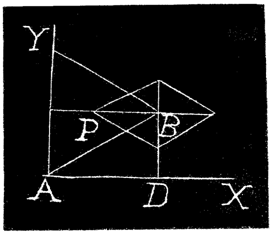
167. Proposed by JOHN M. QUINN, Professor of Mathematics. High School, Warren, Pa.

If at the vertex of an isosceles triangle ne of whose basal vertices is pivoted and the other free to move in a straight line, a rhombus be pivoted with sides parallel to the sides of the triangle, the locus of every point on the rhombus except the one which is its intersection with the fixed side of the triangle is an ellipse.

Solution by the PROPOSER.

Notation. Let X and Y be rectangular axes; θ the angle BAX; m equal segments of the sides of the triangle and the rhombus; $AB=a$, $BD=y$, and $AD=x$, and YBA an isosceles triangle with one basal vertex pivoted at A. The other basal vertex is free to move along the line AY.



To prove that the locus of any point as P is an ellipse.

PROOF. $y/a=\sin\theta$. $\therefore$ $y^2/a^2=\sin^2\theta$.

$x=a\cos\theta-2m\cos\theta=\cos\theta(a-2m)$.

$$\frac{x}{a-2m}=\cos\theta,\ \frac{x^2}{(a-2m)^2}=\cos^2\theta.$$

$$\therefore \frac{y^2}{a^2}+\frac{x^2}{(a-2m)^2}=\sin^2\theta+\cos^2\theta=1.$$ $\therefore$ the locus of p is an ellipse.

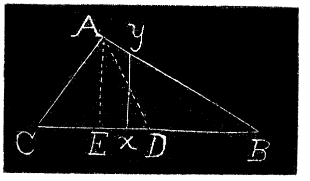
Similarly for any other point *mutatis mutandis*. Q. E. D.

168. Proposed by MISS GUBELMAN, Student Southern Illinois State University, Carbondale, Ill.

To draw a perpendicular to one side of a triangle dividing it into two equivalent parts.

Solution by the PROPOSER.

1. Let ABC be the triangle. Draw the median AD and the perpendicular AE. Construct BX such that $BX^2=BD.BE$. Draw the perpendicular XY.



$\therefore$ $\triangle BXY:\triangle BAE=BX^2:BE^2$.

Similar triangles$=BD\times BE=BE^2=BD:BE$.

Also $\triangle BAD:\triangle BAE=BD:BE$, having equal altitudes.

$\therefore$ $\triangle BXY:\triangle BAE=\triangle BAD:\triangle BAE$.

$\therefore$ $\triangle BXY=\triangle BAD$. But $\triangle BAD=\frac{1}{2}\triangle ABC$. Median.

$\therefore$ $\triangle BXY=\frac{1}{2}\triangle ABC$. $\therefore$ XY is the required perpendicular.

Also solved by *G. B. M. ZERR*, *DANIEL B. NORTHRUP*, *L. C. WALKER*, *J. SCHEFFER*, *H. C. WHITAKER*, *H. B. PENHOLLOW*, and *P. W. WEBBER*.

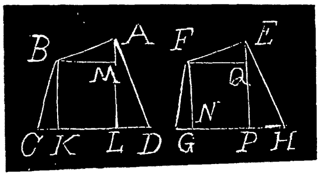
169. Proposed by S. F. NORRIS, Professor of Astronomy and Mathematics, Baltimore City College, Baltimore, Md.

Theorem. Two quadrilaterals having three sides of the one equal to the thre corresponding sides of the other, each to each, and the two corresponding angles adjacent to the unknown sides equal, each to each, are equal figures. [From *Olney's Geometry*, Section VIII, Proposition XIV].

1. Required proof. 2, Is this proposition found in any other text-book of Geometry?

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.

Let $ABCD$, $EFGH$ be the two quadrilaterals; $BC=FG, AB=EF, AD=EH$, $\angle BCD=\angle FGH$, $\angle ADC=\angle EHG$. Draw BK, AL perpendicular to CD; FN, EP perpendicular to GH; BM perpendicular to AL; FQ perpendicular to EP.



Right triangles BCK and FGN are equal, also right triangles ALD and EPH, having hypotenuse and acute angle of one equal to hypotenuse and acute angle of other.

$\therefore$ $BK=FN$, $AL=EP$, also $AL-BK=AM=EP-FN=EQ$.

$\therefore$ right triangles ABM=right triangle FEQ; since $AB=FE$ and $AM=EQ$,

$\therefore$ $BM=FQ$. $\therefore$ $BM=KL=FQ=NP$. $\therefore$ $BKLM=FNPQ$.

$\therefore$ $BCK+ADL+ABM+BKLM=FGN+EPH+FEQ+FNPQ$.

$\therefore$ $ABCD=EFGH$.

Also solved by *J. SCHEFFER*.

170. Proposed by CHARLES C. CROSS, Whaleyville, Va.

If p, q, r are the distances of the orthocenter from the sides, prove that

$$4\left[\frac{a}{p}+\frac{b}{q}+\frac{c}{r}\right]=\left[\frac{a}{p}+\frac{b}{q}-\frac{c}{r}\right]\left[\frac{a}{p}-\frac{b}{q}+\frac{c}{r}\right]\left[-\frac{a}{p}+\frac{b}{q}+\frac{c}{r}\right].$$

Solution by MARCUS BAKER, U. S. Geological Survey, Washington, D. C.

From well known theorems we have

$$\begin{array}{lcl} a=2R\sin A & & p=2R\cos B\cos C \\ b=2R\sin B & \text{and} & q=2R\cos C\cos A \\ c=2R\sin C & & r=2R\cos A\cos B \end{array}$$

Whence $a/p=\tan B+\tan C$, $b/q=\tan C\tan A$, $c/r=\tan A+\tan B$.

Therefore, $a/p+b/q+c/r=2(\tan A+\tan B+\tan C)$.

$$-a/p+b/q+c/r=2\tan A$$

$$a/p-b/q+c/r=2\tan B$$

$$a/p+b/q-c/r=2\tan C.$$

Since $\tan A+\tan B+\tan C=\tan A\tan B\tan C$, the theorem is proved.

Also demonstrated by *L. C. WALKER*, *J. SCHEFFER*, *H. C. WHITAKER*, *G. B. M. ZERR*, and the *PROPOSER*.

171. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

Find the nearest distance of the parabola $y^2=16x$ and the ellipse $16x^2+9y^2-160-144y+832=0$.